MBMT Algebra Round — Zermelo

May 21, 2022

Full Name _____

Student ID Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

_____ **1** Michelle finds a website which gives her a free textbook every day. After gaining 10 textbooks in 10 days, she has tripled her textbook collection. How many textbooks does Michelle have 20 days after she found the website?

_____ **2** Steven really likes palindromes. Palindromes are numbers that read the same backwards and forwards, like 55 or 969. He's thinking of a 3 digit palindrome where the sum of digits is 16 and the ones digit is 5 more than the tens digit. What is the number?

_____ **3** Gablin and Babblin start with different amounts of grapes. If Gablin gives Babblin 1 grape, Gablin would have the number of grapes Babblin has, squared. If Babblin gave Gablin 1 grape instead, Gablin would have had the number of grapes Babblin has, cubed. How many grapes does Gablin have?

_____ **4** Find the number of integers whose nearest perfect square is 264^2, including 264^2 itself.

_____ **5** What fraction of real numbers between 0 and 2 satisfy $\lceil x - \frac{2}{15} \rceil = \lfloor x + \frac{2}{15} \rfloor$? Here, $\lceil x \rceil$ is the smallest integer greater than or equal to x, and $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

_____ **6** For some constant value(s) of c, the following system of equations has infinite solutions.

$$\begin{cases} x = |y - 4| - 20 \\ y = |x - c| \end{cases}$$

What is the sum of all possible values of c?

_____ **7** Given real x, y, and z such that $x + y = 4$ and $x^2 + y^2 + xy + xz + yz = 288$, find the maximum possible value of z.

_____ **8** Let $x_0 = 4$ and $x_{n+1} = x_n + 2\sqrt{x_n - 1} + 1$ for $n \geq 0$. What is x_{100}?