# MBMT Descartes Guts Round — Set 1

**March 30, 2019**

____________ **1** **[3]** Find the units digit of $3^{1^{3^{3^7}}}$.

____________ **2** **[3]** Find the positive solution to the equation $x^3 - x^2 = x - 1$.

____________ **3** **[3]** Points $A$ and $B$ lie on a unit circle centered at $O$ and are distance 1 apart. What is the degree measure of $\angle AOB$?

____________ **4** **[3]** A number is a perfect square if it is equal to an integer multiplied by itself. How many perfect squares are there between 1 and 2019, inclusive?

____________ **5** **[3]** Ted has four children of ages 10, 12, 15, and 17. In fifteen years, the sum of the ages of his children will be twice Ted's age in fifteen years. How old is Ted now?

# MBMT Descartes Guts Round — Set 2

**March 30, 2019**

____________ **6** **[4]** Mr. Schwartz is on the show Wipeout, and is standing on the first of 5 balls, all in a row. To reach the finish, he has to jump onto each of the balls and collect the prize on the final ball. The probability that he makes a jump from a ball to the next is $1/2$, and if he doesn't make the jump he will wipe out and no longer be able to finish. Find the probability that he will finish.

____________ **7** **[4]** Kevin has written 5 MBMT questions. The shortest question is 5 words long, and every other question has exactly twice as many words as a different question. Given that no two questions have the same number of words, how many words long is the longest question?

____________ **8** **[4]** Square $ABCD$ with side length 1 is rolled into a cylinder by attaching side $AD$ to side $BC$. What is the volume of that cylinder?

____________ **9** **[4]** Haydn is selling pies to Grace. He has 4 pumpkin pies, 3 apple pies, and 1 blueberry pie. If Grace wants 3 pies, how many different pie orders can she have?

____________ **10** **[4]** Daniel has enough dough to make 8 12-inch pizzas and 12 8-inch pizzas. However, he only wants to make 10-inch pizzas. At most how many 10-inch pizzas can he make?

# MBMT Descartes Guts Round — Set 3

**March 30, 2019**

______ **11** **[5]** A standard deck of cards contains 13 cards of each suit (clubs, diamonds, hearts, and spades). After drawing 51 cards from a randomly ordered deck, what is the probability that you have drawn an odd number of clubs?

______ **12** **[5]** Let $s(n)$ be the sum of the digits of $n$. Let $g(n)$ be the number of times $s$ must be applied to $n$ until it has only 1 digit. Find the smallest $n$ greater than 2019 such that $g(n) \neq g(n+1)$.

______ **13** **[5]** In the Montgomery Blair Meterology Tournament, individuals are ranked (without ties) in ten categories. Their overall score is their average rank, and the person with the lowest overall score wins.

Alice, one of the 2019 contestants, is secretly told that her score is $S$. Based on this information, she deduces that she has won first place, without tying with anyone. What is the maximum possible value of $S$?

______ **14** **[5]** Let $A$ and $B$ be opposite vertices on a cube with side length 1, and let $X$ be a point on that cube. Given that the distance along the surface of the cube from $A$ to $X$ is 1, find the maximum possible distance along the surface of the cube from $B$ to $X$.

______ **15** **[5]** A function $f$ with $f(2) > 0$ satisfies the identity $f(ab) = f(a) + f(b)$ for all $a, b > 0$. Compute $\frac{f(2^{2019})}{f(2^3)}$.

# MBMT Descartes Guts Round — Set 4

**March 30, 2019**

__________ **16** **[7]** Alex has 100 Bluffy Funnies in some order, which he wants to sort in order of height. They're already *almost* in order: each Bluffy Funny is at most 1 spot off from where it should be. Alex can only swap pairs of adjacent Bluffy Funnies. What is the maximum possible number of swaps necessary for Alex to sort them?

__________ **17** **[7]** I start with the number 1 in my pocket. On each round, I flip a coin. If the coin lands heads heads, I double the number in my pocket. If it lands tails, I divide it by two. After five rounds, what is the expected value of the number in my pocket?

__________ **18** **[7]** Point $P$ inside square $ABCD$ is connected to each corner of the square, splitting the square into four triangles. If three of these triangles have area 25, 25, and 15, what are all the possible values for the area of the fourth triangle?

__________ **19** **[7]** Mr. Stein and Mr. Schwartz are playing a yelling game. The teachers alternate yelling. Each yell is louder than the previous and is also relatively prime to the previous. If any teacher yells at 100 or more decibels, then they lose the game. Mr. Stein yells first, at 88 decibels. What volume, in decibels, should Mr. Schwartz yell at to guarantee that he will win?

__________ **20** **[7]** A semicircle of radius 1 has line $\ell$ along its base and is tangent to line $m$. Let $r$ be the radius of the largest circle tangent to $\ell$, $m$, and the semicircle. As the point of tangency on the semicircle varies, the range of possible values of $r$ is the interval $[a, b]$. Find $b - a$.

# MBMT Descartes Guts Round — Set 5

**March 30, 2019**

__________ **21** **[9]** Hungryman starts at the tile labeled "S". On each move, he moves 1 unit horizontally or vertically and eats the tile he arrives at. He cannot move to a tile he already ate, and he stops when the sum of the numbers on all eaten tiles is a multiple of nine. Find the minimum number of tiles that Hungryman eats.

| | | | | |
|---|---|---|---|---|
| S | 7 | 9 | 16 | 18 |
| 25 | 27 | 36 | 45 | 52 |
| 54 | 63 | 70 | 72 | 81 |
| 88 | 90 | 99 | 108 | 115 |
| 117 | 124 | 126 | 133 | 135 |

__________ **22** **[9]** How many triples of nonnegative integers $(x, y, z)$ satisfy the equation $6x + 10y + 15z = 300$?

__________ **23** **[9]** Anson, Billiam, and Connor are looking at a 3D figure. The figure is made of unit cubes and is sitting on the ground. No cubes are floating; in other words, each unit cube must either have another unit cube or the ground directly under it. Anson looks from the left side and says, "I see a $5 \times 5$ square." Billiam looks from the front and says the same thing. Connor looks from the top and says the same thing. Find the absolute difference between the minimum and maximum volume of the figure.

__________ **24** **[9]** Tse and Cho are playing a game. Cho chooses a number $x \in [0, 1]$ uniformly at random, and Tse guesses the value of $x(1 - x)$. Tse wins if his guess is at most $\frac{1}{50}$ away from the correct value. Given that Tse plays optimally, what is the probability that Tse wins?

__________ **25** **[9]** Find the largest solution to the equation

$$2019(x^{2019x^{2019}-2019^2+2019})^{2019} = 2019^{x^{2019}+1}.$$

# MBMT Descartes Guts Round — Set 6

**March 30, 2019**

**This round is an estimation round. No one is expected to get an exact answer to any of these questions, but unlike other rounds, you will get points for being close. In the interest of transparency, the formulas for determining the number of points you will receive are located on the answer sheet, but they aren't very important when solving these problems.**

**To receive points, your answers should be positive and in decimal notation. For example, $\mathbf{10.55}$ is allowed, but not $\mathbf{-3.2}$ or $\frac{2\pi}{3}$.**

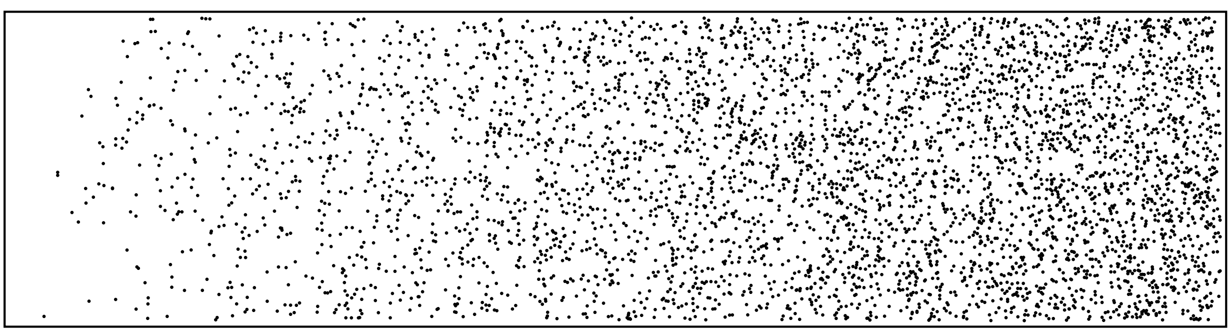
________ **26** **[12]** What is the sum over all MBMT volunteers of the number of times that volunteer has attended MBMT (as a contestant or as a volunteer, including this year)?

Last year there were 47 volunteers; this is the fifth MBMT.

________ **27** **[12]** William is sharing a chocolate bar with Naveen and Kevin. He first randomly picks a point along the bar and splits the bar at that point. He then takes the smaller piece, randomly picks a point along it, splits the piece at that point, and gives the smaller resulting piece to Kevin. Estimate the probability that Kevin gets less than 10% of the entire chocolate bar.

________ **28** **[12]** Let $x$ be the positive solution to the equation $x^{x^{x^x}} = 1.1$. Estimate $\frac{1}{x-1}$.

________ **29** **[12]** Estimate the number of dots in the following box:



It may be useful to know that this image was produced by plotting $(4\sqrt{x}, y)$ some number of times, where $x, y$ are random numbers chosen uniformly randomly and independently from the interval $[0, 1]$.

________ **30** **[12]** For a positive integer $n$, let $f(n)$ be the smallest prime greater than or equal to $n$. Estimate

$$(f(1) - 1) + (f(2) - 2) + (f(3) - 3) + \cdots + (f(10000) - 10000).$$

For $26 \leq i \leq 30$, let $E_i$ be your team's answer to problem $i$ and let $A_i$ be the actual answer to problem $i$. Your score $S_i$ for problem $i$ is given by

$$\begin{aligned}
S_{26} &= \max(0, 12 - |E_{26} - A_{26}|/5) \\
S_{27} &= \max(0, 12 - 100|E_{27} - A_{27}|) \\
S_{28} &= \max(0, 12 - 5|E_{28} - A_{28}|)) \\
S_{29} &= 12 \max\left(0, 1 - 3\frac{|E_{29} - A_{29}|}{A_{29}}\right) \\
S_{30} &= \max(0, 12 - |E_{30} - A_{30}|/2000)
\end{aligned}$$