MBMT Number Theory Round — Gauss

April 7, 2018

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

_____ **1** Haydn is playing with his toy cars. He has between 20 and 30 cars, and he knows that when he tries to put them into groups of 12, he has 1 left over. How many toy cars does Haydn have?

_____ **2** How many 2 digit numbers $\overline{AB}$ are there so that both $\overline{AB}$ and A are divisible by 3? (If $A = 1$ and $B = 7$, then $\overline{AB} = 17$.)

_____ **3** For how many ordered pairs (a, b), where a and b are positive integers, is the value $2^a 8^b$ less than 1000?

_____ **4** The least common multiple of two natural numbers is 8 times their greatest common factor. What is the value of the larger number divided by the smaller number?

_____ **5** Every time DJ Khaled says "another one" he either adds a 1 to his number or appends a 1 to the end of his number. For example, he can turn 6 into 7 or 61. If he starts with the number 1, what is the minimum number of turns to get to 2018?

_____ **6** How many positive integers n less than 1000 satisfy the property that $\lfloor \sqrt[3]{n} \rfloor$ is a factor of n? Here, $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

_____ **7** What is the smallest positive integer n such that n divided by 7 has remainder 3, n divided by 11 has remainder 5, n divided by 13 has remainder 6, and n divided by 17 has remainder 8?

_____ **8** Find

$$\gcd(2^{71} - 2, 3^{71} - 3, \ldots, 100^{71} - 100).$$