MBMT Number Theory Round — Cantor

April 7, 2018

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

1 Dilcue's favorite number is a one-digit positive integer. When she multiplies her favorite number by 4, the result is a two-digit number that ends in 8. What is 5 times Dilcue's favorite number?

2 Anna has a perfect square number of marbles. Bob has a perfect cube number of marbles. Both have more than one marble and fewer than ten marbles. Bob has one fewer marble than Anna. How many do they have altogether?

Perfect squares are squares of integers: 0, 1, 4, etc. Perfect cubes are cubes of integers: 0, 1, 8, etc.

3 What is the smallest number greater than 22 that can be written as a sum of two perfect squares?

4 How many 2 digit numbers $\overline{AB}$ are there so that both $\overline{AB}$ and A are divisible by 3? (If $A = 1$ and $B = 7$, then $\overline{AB} = 17$.)

5 For how many ordered pairs (a, b), where a and b are positive integers, is the value $2^a 8^b$ less than 1000?

6 The least common multiple of two natural numbers is 8 times their greatest common factor. What is the value of the larger number divided by the smaller number?

7 Guang has come up with a great time management strategy. He decides that every 91 days he will rap continuously for the entire day. Some time later, he also decides that every k days (where k is an integer) he will do math for the entire day. Assuming that Guang cannot rap and do math simultaneously, what is the minimum k for which this is possible?

8 What is the smallest positive integer n such that n divided by 7 has remainder 3, n divided by 11 has remainder 5, and n divided by 13 has remainder 6?